RESULTS ON SOME CONTRACTIONS AND NONEXPANSIVE MAPPINGS WITH APPLICATIONS

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ABSTRACT

The metric fixed point theory acts as an effective tool in applied mathematics, physical sciences, engineering, economics, biology and many other correlated subjects. Inspired by the growing influence of this literature, in this discourse we come up with several exciting notions and results which generalize, complement, improve and certainly consolidate numerous existing ones.

In this disquisition, we propose the notions of asymptotic contractions for a pair of mappings and for a finite family of mappings, and subsequently that of for weak asymptotic contractions. Then we derive a few pertinent common fixed point results using these asymptotic contractions in pair concepts. Furthermore, we also enquire for a couple of common fixed point results involving the newly introduced enriched contractions in pair in the metric spaces setting.

On the application point of view, we implement our derived results in the context of asymptotic contractions to attain some sufficient conditions for existence of the unique common solution to a certain kind of pair of nonlinear matrix equations. Besides, we conceive a new iterative algorithm in the context of enriched contractions to get to the unique common positive definite solution to a pair of nonlinear matrix equations. Finally, we set up some suitable numerical examples to illustrate our results by using figurative presentations.

Besides, we claim that the lately originated $\mathscr{F}$-metric space is metrizable. Also, we derive that the concepts of convergence, Cauchy sequence, completeness due to Jleli and Samet concerning $\mathscr{F}$-metric spaces are analogous to those of usual metric spaces. Moreover, we affirm that the Banach contraction principle in $\mathscr{F}$-metric spaces context is a direct consequence of its standard metric counterpart.

Finally, we affirm some exciting results concerning the newly introduced generalized α-Reich-Suzuki non-expansive mappings in the context of Banach spaces. Also we establish the three step Abbas-Nazir iterative scheme for a pair of mappings and we derive a necessary and sufficient condition to obtain the existence of common fixed points of two generalized α-Reich-Suzuki nonexpansive mappings. Further, we enquire for a couple of weak and strong convergent results using our newly introduced Abbas-Nazir iterative scheme for the aforementioned kind of mappings. Besides, in hyperbolic metric spaces context, we study some convergence results and also,

the approximation of the fixed points for a generalized α-Reich-Suzuki nonexpansive mapping using the newly proposed JF-iteration scheme.

Eventually, we furnish a number of constructive nontrivial numerical examples to authenticate our attained findings. We have used MATLAB 2017a software for the examples related to generalized α-Reich-Suzuki nonexpansive mappings to compare the rate of convergence of the implemented iteration schemes with that of some other well-known algorithms.

Keywords: Asymptotic contraction, common fixed point, matrix equation, positive definite matrix, enriched contraction, $\mathscr{F}$-metric, metrizability, Banach space, nonexpansive mapping, uniformly convex Banach space, Opial property, Fréchet differentiable, trace norm, iterative algorithm, demiclosednes, weak convergence, Δ-convergence, condition (I).